

Fraction arithmetic

This document should help you to understand why fractions are multiplied and divided using the rules given in Subsection 3.3 of MST124 Unit 1. It's an edited extract from the module MU123 *Discovering mathematics*.

First let's look briefly at how to multiply fractions. Consider the following example. Suppose that three-quarters of the three-year-olds in a village attend a nursery, and half of them attend full-time. What fraction of the toddlers attend nursery full-time? The answer is a half of three-quarters, and you can see from Figure 1 that this is three-eighths. That is,

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}.$$

This is an example of the following general rule.

Strategy To multiply fractions

Multiply the numerators together and multiply the denominators together.

Now let's look at how to divide fractions. The rule for doing this can be conveniently described using the idea of the *reciprocal* of a number.

Remember that a number and its **reciprocal** multiply together to give 1. Another way to think of the reciprocal of a number is that it is 1 divided by the number. To find the reciprocal of a fraction, you swap the numerator and denominator. For example,

the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

the reciprocal of $\frac{1}{4}$ is $\frac{4}{1} = 4$

the reciprocal of 2 is $\frac{1}{2}$, since $2 = \frac{2}{1}$.

Now suppose that you have a length of string, and you plan to cut it into two-metre pieces. How many pieces will you get? The answer is the length of the string in metres, divided by 2.

Next suppose that you want to cut the string into pieces one-third of a metre long. How many pieces will you get this time? The answer is the length of the string in metres, divided by $\frac{1}{3}$. But how do you divide by $\frac{1}{3}$?

Well, you get three pieces for every metre of string (Figure 2), so you need to multiply the length of the string by 3. So dividing by $\frac{1}{3}$ is the same as multiplying by 3, the reciprocal of $\frac{1}{3}$.

What if you want to cut the string into pieces two-thirds of a metre long – how many pieces will you get this time (Figure 3)? That is, how do you divide by $\frac{2}{3}$? Well, the number of pieces two-thirds of a metre long is half of the number of pieces one-third of a metre long. So to divide by $\frac{2}{3}$ you multiply by 3 and then by $\frac{1}{2}$, which is the same as multiplying by $\frac{3}{2}$. So dividing by $\frac{2}{3}$ is the same as multiplying by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

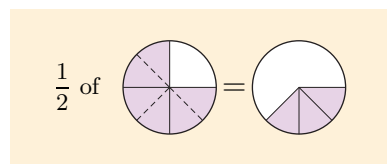


Figure 1 $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

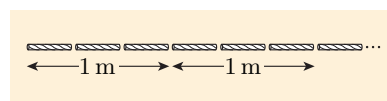


Figure 2 A length of string cut into one-third metre pieces

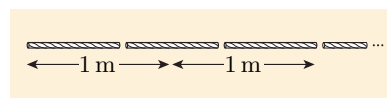


Figure 3 A length of string cut into two-third metre pieces

These are examples of the following general rule.

Strategy *To divide by a fraction*
Multiply by its reciprocal.